The Reality of Length Contraction and Time Dilation

Time Dilation

That rapid motion and strong gravitational fields cause clocks to slow down is an experimentally verified fact. Muons reaching the Earth's surface from the upper atmosphere decay at a slower rate than they should and atomic clocks carried by GPS satellites have to be corrected for the effects of gravity. These effects are real enough – but are we required to look for a *physical reason* why clocks in motion or at the bottom of a well run slowly? Or is it just that *time itself* runs slowly?

Length Contraction

As far as I know, the phenomenon of length contraction has never been experimentally demonstrated but in so far as it is a logical consequence of time dilation and the constancy of the speed of light (a fact which is verified by the Michelson-Morley experiment) we must regard length contraction as a fact also. But does the motion of a space ship actually cause the ship to contract *physically?* Or is it just that it *looks like* this from our stationary point of view? Or it is that *space itself* contracts?

Three crucial thought experiments

In discussing these issues, three thought experiments are useful. The first is the famous Twins Paradox. Albert travels to Alpha Centauri which is 4 light years away from Earth at 80% of the speed of light. At this speed the relativistic gamma factor $\gamma = \frac{1}{\sqrt{1 - v^2/c^2}}$ equals 5/3. From Albert's point of view, the distance to the star has been contracted to 4 × 3/5 = 2.4 light years and at 80% of the speed of light, he will get to the star after 3 years travel time. It will take him a further 3 years to get back so when he returns to Earth he will be 6 years older than when he left. His twin brother Ludvig, however, sees Albert travel at 80% of the speed of light so, according to him, it will take Albert 5 years to get there and 5 years to get back and so Ludvig will be 10 years older when his brother returns. In spite of much debate over the decades, nobody now seriously doubts the logic of the above analysis and the outcome is accepted as fact. It is also agreed that the reason why it is Albert who is younger than Ludvig and not the other way around is because it is Albert who changes from one inertial frame to another when he turns round at his destination. What I wish to discuss here is whether we should look for some *physical reason* why Alberts clocks should run more slowly than Ludvig's or whether we should talk about *time running more slowly* for Albert than it does for Ludvig.

The second crucial experiment is closely related. Albert and Ludvig synchronise their watches; then Albert spends 10 years (of Ludvig's time) doing experiments down a deep mine shaft where the relativistic gamma factor $\gamma = \frac{1}{\sqrt{1 - 2\Delta\phi/c^2}}$ equals 1.1 ($\Delta\varphi$ is the gravitational potential difference between the top and the bottom of the mine shaft). What this means is that when Albert returns to the surface he will only be 10/1.1 = 9.09 years older and hence he will emerge nearly a year younger than his brother.. Again, the facts are not at issue. The question is whether we should look for a *physical reason* which causes Albert's clock to run slow – or is it that *time itself runs more slowly* at the bottom of a mine shaft than at the top or is it something else entirely?

The third crucial experiment is called the broken rope paradox or, alternatively, Bell's spaceship paradox. Two spaceships at rest and 100 km apart in Ludvig's frame are tethered together with a rope 100 km long. Ludvig stands halfway between them and causes a flash of light. At the instant that the flash of light reaches the two spaceships (and therefore simultaneously in Ludvig's frame) the spaceships accelerate rapidly with identical acceleration. At all times (in Ludvig's frame) the two spaceships remain 100 km apart – but the rope, once is is moving at a sizeable fraction of the speed of light, is contracted in length and must either stretch of break. But why? Does this prove that length contraction is a real physical phenomenon? Or is there some other explanation involving different points of view and/or the contraction of space?

Two opposing schools of thought

Many introductory books on Relativity concentrate on Special Relativity and base their accounts on the Lorentz transformations which allow one to translate the coordinates on any event in one frame of reference into coordinates in another. The implication is that two observers in relative motion simply have a different perspective on the reality which is being played out before them. When Ludvig 'sees' Albert flash past him in a shortened spaceship, it appears to be shortened to him simply because of his point of view, just as a circular table appears to be foreshortened into an ellipse when viewed obliquely. We might call these authors 'illusionists' because they claim that length contraction is just an illusion caused by looking at reality from a certain perspective.

There is much to be said for this argument. In engineering, complex objects are often specified in three dimensions using what is known as first (or third) angle projection in which the object is shown in plan and elevation from two sides. None of the three views give the full picture because they are two dimensional – but taken together, they are sufficient to describe a three dimensional object. In the same way, it is argued, neither Ludvig's nor Albert's perspective is sufficient to describe completely what is going on; you need both to get the full picture.

I don't buy this. Firstly, it is sometimes implied that there is one perspective which is more important than any of the others (eg Ludvig's 'inertial' or 'stationary' perspective). We should reject this idea. All perspectives should be regarded as equally valid. Secondly, the question arises – how many different perspectives are needed to 'get the full picture'? In engineering, three perspectives are sufficient because the reality it is describing is three dimensional. Events in spacetime are four dimensional; do we need to invoke a 5 or more dimensional reality which we each view in four dimensions from our different perspectives? Possibly, but I do not recommend going down that road. But if we accept that every perspective is equally valid and that there is no higher dimension or ultimate reality, then we must accept that different observers must explain what they see in front of them in different ways – the only serious constraint being that all the different observers must use the same laws of Physics to derive their individual explanations. This is what I shall call the 'realist' position.

I shall now try to explain the three crucial thought experiments mentioned above according to the realist school of thought in reverse order.

The Broken Rope Paradox

The fact is: the rope breaks. Why?

In Ludvig's frame of reference, the two spaceships stay the same distance apart all the time but as the rope moves faster and faster, it contracts and breaks. The realist must put forward a physically correct explanation for the contraction. The explanation is this. The forces of interaction between atoms in a material are essentially electromagnetic in nature and are determined by Maxwell's laws of electromagnetism. Now Maxwell's law are what is known as Lorentz invariant. What this means is that they are already in a form which is compatible with Special Relativity. Indeed, it was the very fact that Maxwell's laws effectively imply the constancy of the speed of light which gave rise to the whole theory in the first place. Now when electric charges are in motion, you have to take *magnetic* forces into account as well as electrostatic ones. This will deform the electron orbitals round the atoms and cause the rope physically to shrink. (For a more detailed defence of this position see J. S. Bell: *How to teach special relativity*, in *Speakable and Unspeakable in Quantum Mechanics* (Cambridge University Press, Cambridge, 1997), pp. 67-80. or N. David Mermin: *It's About Time* (Princeton University Press, 2005) pp179-186)

Now consider the situation from Albert's point of view. Albert is moving in the same direction as the spaceships and at the same ultimate velocity v. From his point of view, the two tethered spaceships are approaching him backwards ate a speed v and, of course, he notes that the spaceships are less than 1 km apart because, to him, the separation of the spaceships (and the rope) has already been Lorentz contracted. As he flies by, the two spaceships fire up their rockets and eventually come to rest beside him (because now, Albert, the two ships and the rope are all travelling at the same speed in Ludvig's frame). From Albert's point of view, the rope will now *expand*. Why then does the rope break from his point of view? The answer is simple. In his frame of reference, the leading spaceship starts to move *before* the trailing one because the flash of light reaches the leading spaceship first. Both Albert and Ludvig agree that the rope breaks – but they have different explanations for the result.

Now you might very well object to any physical explanation of length contraction on the following grounds. If length contraction really depends on the nature of the electromagnetic forces between atoms, why does the contraction depend solely on the speed of motion? You might expect. For example, a steel rod to shrink differently from a wooden one. Surely it is too much to expect all materials to shrink in exactly the same way? In any case, might it not be possible to imagine a rope made, for example, from a string of neutrons bound together by the strong nuclear force rather than by electromagnetic forces? Good try. But it doesn't matter. Even the strong and weak nuclear forces are *Lorentz invariant* and therefore even a string of neutrons would shrink in exactly the same way. It is the laws of physics which cause the shrinking and the laws of physics are the same whether the rope is made of steel, wood or neutrons.

If you were really persistent, you might carry your objection even further – after all, this is just a thought experiment. Suppose, you say, that the rope was made of a series of fairies of zero mass joined together with inextensible gossamer threads which used neither electromagnetic forces or anything else to ensure that the fairies stay the same distance apart. Would the rope break then? No, of course it would not. What you are really describing is a set of coordinate points attached to the two spaceships. When the spaceships start to move, they carry their coordinate system with them.

To summarise: Length contraction is a real physical effect and has a physical cause. It does not make sense to talk about *space* contracting because space is just a coordinate system; if you define two points to be 1 metre apart, you cannot then turn round and say that they are now 2 metres apart; and if you point out that it is now necessary to use two metre rulers to stretch from one point to the other, you must conclude that the rulers have shrunk, not that the space has expanded.

The clock at the bottom of the mine

Now what about the clock at the bottom of the mine. Compared to the clock at the top, it runs slow. This is a fact. Why?

The first thing to get out of the way is that the clock does not run slow because gravity is stronger at the bottom of a mine. This is a common and serious mistake[1]. The clock will run slow even if the gravitational field is uniform. It is therefore emphatically not the case that the clock is responding to some local effect because both clocks are situated in identical regions. The thing that makes them differ is the fact that one clock is situated above the other and energy is needed to go

1 Even Martin Gardner is guilty of this error. On page 116 of his otherwise excellent book 'Relativity Simply Explained' he says: "Experiments … have shown that time near the bottom of a building (where gravity is stronger) is a bit slower than time near the top."

from one to the other.

Now at first glance, this seems to rule out the idea that the slowing of clocks in a gravitational field cannot possibly have anything to do with physical processes in the same way that motion can have an effect on the forces between electric charges. And you are right. *Gravity does not, in itself, slow down clocks*.

So what does gravity do?

Gravity curves spacetime.

Now I am not going to attempt to explain what this means. Very few books have been written which even try and even fewer have been successful. Perhaps the best attempt is Archibald Wheeler's *Journey into Gravity and Spacetime* and a unique approach is to be found in *Relativity Visualized* by Lewis Carroll Epstein. But the effects of the curvature of spacetime on clocks and rulers is now well understood, mathematically, and the likes of you and me will just have to accept the computed results.

It is quite easy to show why the flashes of light emitted at 1 second intervals by a clock at the back of an *accelerating* spaceship reach the front of the spaceship more than 1 second apart. (This is effectively just the Doppler effect in light). It is therefore very tempting to say that a clock at the back of a spaceship *runs more slowly* than a clock at the front. But this would be a mistake. Both clocks are identical and are subject to identical conditions. Both clocks tick 1 second every second. So is it correct to say that *time runs more slowly* at the back of the spaceship than at the front? This would be a mistake too (though it is a very useful fiction which I am guilty of using many times myself).

So what is the solution? The answer is that, gravity does not cause clocks or time to run slow, it causes a *Doppler shift* in the signals which travel from the bottom of the mine to the top. But surely, you may argue, when the clocks are reunited, one clock is slower than the other so that proves that the one at the bottom of the mine must have been running more slowly, doesn't it? No it doesn't. If you are running a marathon and you find that your elderly father has got to the finish before you – does that prove that he ran faster than you? Not at all. It only goes to show that he must have taken a short cut along the way!

To summarise: We have a semantic problem here. We cannot say that time runs slowly at the bottom of a mine because time always runs at a rate of 1 second per second. We cannot say that a clock at the bottom of a mine runs more slowly than a clock at the top because the two clocks are physically identical. So what can we say? We have the same problem describing distances on Earth. At the equator, each degree of longitude represents 67 miles on the ground – but at the latitude of London, one degree of longitude corresponds to only 43 miles. We can't say that the Earth is squashed in the region of London – 1 mile is still 1 mile – and it is silly to say that our longitude rulers expand when we take them North. The effect is entirely due to the fact that the surface of the Earth is curved. In the same way, the phenomenon of gravitational Time Dilation is entirely due to the curvature of spacetime. Two explosions A and B occur 67 seconds apart at the top of the mine; two events A' and B' occur at the bottom of the mine simultaneous with A and B (or at any rate at the same temporal interval later e.g. when the sound of the explosions reaches them); if A' and B' are 43 seconds apart this is simply because spacetime is curved, not because time runs slowly at the bottom of a mine or that gravity slows down clocks. Nothing more need be said. In the absence of any suitable phraseology to describe this effect we often resort to the convenient but potentially misleading *fiction* that time runs slowly at the bottom of a mine compared to time at the top, but we definitely do not have to search for a physical reason to explain why clocks appear to run more slowly. When a clock at the bottom of a mine is reunited with a clock at the top they do not agree but this is simply because they have each *taken a different journey through spacetime*.

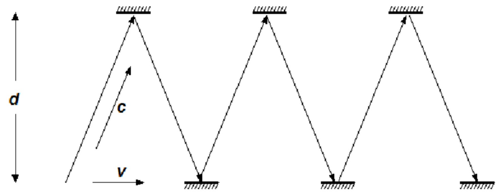
The Twins Paradox

The question at issue here is – does motion, in itself, cause clocks to run more slowly?

From what I have said in the last paragraph, you will perceive the I am going to reject that idea that motion makes *time* run more slowly. It makes no sense to speak of the rate at which time runs. That just leads into an infinite regress. But is there a physical reason why *clocks* should run more slowly when they are in motion?

Yes there is. It might be difficult to explain why pendulum clock or quartz watch runs slow when in motion but there is a very simple kind of clock which shows the principle very clearly. It consists of a pair of parallel mirrors a distance 1 light second apart with a beam of light bouncing back and forth. Each time the light beam hits one of the mirrors the clock gives out a tick and advances its counter by 1 second.

Now suppose that the apparatus is moved at a speed v in a direction at right angles to the perpendicular distance between the mirrors. (This is to avoid complications involving length contraction because the perpendicular distance will not be affected by the motion.) Now the path of the light beam is a zig-zag like this:



A simple light clock

It is easy to prove that the actual length of the distance travelled between ticks is going to be multiplied by a factor $\frac{1}{\sqrt{1 - v^2/c^2}}$ and that the clock will therefore run more slowly by this factor. It does not matter whether the clock moves to the right or to the left, the zig-zag distance is always increased so you can, if you like, move the clock to the right for a while and then back again to compare it with a similar clock which remains stationary. The moving clock will record fewer ticks than the stationary one. Moving clocks run slow.

I admit that it is not easy to see how the physics which causes this simple clock to run slow can be translated into the physics which causes atomic clocks, quartz clocks, pendulum clocks and biological clocks to run slow but it *must* be so. All interactions between atoms involve the exchange of photons and if these photons have further to travel, then the whole process must proceed more slowly.

This is also true of clocks which do not depend on electromagnetism. It is well known that the half life of muons generated in the upper atmosphere is increased by their motion and similar effects which depend solely on the weak and strong nuclear forces are daily observed in the chambers of the accelerators at Fermilab and CERN. Once we have elevated the idea that it is impossible by any local experiment to detect motion through absolute space to the status of a fundamental principle on a par with the fundamental assumptions of isotropy and homogeneity, then all the laws of physics including quantum mechanics must be couched in a Lorentz invariant form.

To summarise: Motion *physically causes all clocks however they are constructed to run slow.* It makes no sense to say that motion causes *time* to run slow.

But the Twins Paradox has more to teach us.

The real paradox of the twins

The real paradox of the twins is not that the motion causes Albert to age more slowly than his brother Ludvig, but the fact that all motion is relative seems to imply that you can regard Albert as stationary and Ludvig who moves – in which case it ought to be Ludvig's clocks which run slow, not Albert's.

The conventional explanation is that since it is Albert who stops and turns round, the situation is not symmetrical. When Albert turns round he steps off one frame of reference and steps onto another. It is true that, during the 3 years which Albert takes to travel out to the star, he perceives his brother moving away from him at 80% of the speed of light and he may, correctly, infer that during this time, his brother's clock only advances 3 × 3/5 = 1.8 years; but this calculation is academic as he cannot possibly compare his own clock with his brother's at this stage.

Now, when he steps off the outgoing frame of reference onto the homeward frame, what he regards as being simultaneous – i.e. 'now' – changes radically. In fact, time back at home suddenly jumps forwards by 6.4 years making time there 1.8 + 6.4 = 8.2 years. Then, during the 3 years of Albert's homeward journey, he imagines Ludvig's clock creeping slowly forward by a further 1.8 years so when they are reunited, Albert is 6 years older and Ludvig is 10 years older!

As they say, it all works out beautifully.

But some author's have been puzzled by the fact that time back at home suddenly jumps forward by 6.4 years when Albert turns round, and many of them have concluded that this must be something to do with the fact that Albert has to suffer some serious deceleration and acceleration during the process of turning round. Many authors (including Martin Gardner) have suggested that these decelerations and accelerations during the turnaround cause Albert's clock to go slow by analogy with clocks at the bottom of a mine – effectively stopping still for 6.4 years – but this is false. As I have said above – *gravity does not cause clocks to run slow.* If it did than Albert would be even younger when he returned than the 6 years predicted by Special Relativity.

And yet, there is the germ of a correct idea in all of this; but the decelerations and accelerations which Albert experiences do not cause Albert's clocks to run slow – in a sense, they cause Ludvig's clocks to run fast!

Let me explain.

All motion is relative. We should, therefore be able to explain what bis going on by assuming that Albert is stationary the whole time. For 3 years of Albert's time, Ludvig departs on the receding Earth at 80% of the speed of light during which time Albert calculates that his clocks will age 1.8 years.

At this point, Albert switches on his retro rockets and decelerates to zero, immediately accelerating back up again to his former speed. During this time he will, of course, experience some massive forces on his body, but we have supposed that he doesn't actually move because we are assuming that he is stationary throughout. How can he interpret these massive forces? He must conclude that someone has switched on a powerful gravitational field g for a short time t. directed towards him from Earth.

Now what this means is that, during this time t, Albert is effectively at the bottom of a mine and that this is the reason why there is a sense in which Ludvig's clocks run fast and skip ahead by 6.4 years.[2]

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2 The idea can be made mathematically respectable too but you cannot use the standard formula for the gravitational time dilation factor $\gamma = \frac{1}{\sqrt{1 - 2gD/c^2}}$ because of the effect the gravitational field has on the distance D between Albert and Earth. In the book '*It's about Time*' by David Mermin the author gives a simple mathematical

Have we proved that General Relativity is needed to explain the Twin's Paradox? No we have not. All we have done is to show that General Relativity is *consistent* with Special Relativity. The conventional explanation is still the better explanation – but it must always be possible to generate a consistent explanation of the actual events using the same laws of physics from any observer's perspective.

One last puzzle

Now you may recall that, earlier, I claimed that Albert gets to Alpha Centauri – a distance of 4 light years away – in only 3 years because from his point of view, the distance has been contracted to only 2.4 light years because of the relativistic contraction. But Earth and Alpha Centauri are not connected together by a rope so how can the physical distance between them be contracted? In discussing the paradox of the broken rope, didn't I say that the distance between the two spaceships stays the same? And since, as I said, all perspectives are equal, can't we assume that Albert is, in fact, stationary and that at the instant he leaves Earth, it is both Earth and Alpha Centauri who suddenly start to move backwards with a speed of 80% of the speed of light? If so, what is it that causes Alpha Centauri and Earth to move closer together?

OK let's assume that Albert is stationary all the time. At time $t = 0$ he switches on his rockets, but since he doesn't move he must conclude that someone has simultaneously switched on an equivalent gravitational field g which is directed from Alpha Centauri towards Earth. Now I am not going to attempt to show mathematically how, at the end of this period Alpha Centauri ends up by being only 2.4 light years away instead of 4 but I am confident that it can, in principle, be done because I know that, at the end of the day, that is what happens. It is not that Alpha Centauri physically moves towards Earth during these few seconds, it is just that *Alpha Centauri and Earth take different routes through curved spacetime* and end up closer together (from Albert's point of view, of course). It is the same as the clock at the bottom of the mine. Both clocks runs at the correct speed all the time but in the presence of a gravitational field they take different routes and end up showing a time difference. Similarly, both Earth and Alpha Centauri remain stationary in space but owing to the gravitational field which Albert experiences in his accelerated frame, they take different routes through spacetime and end up by being closer together.

Concluding remarks

It is probably true to say that more misleading and even false statements have been written on the subject of Special Relativity than any other subject in Physics. (People who write about General Relativity usually know what they are talking about but what they say is often incomprehensible!). The main source of confusion is the firm belief that all the bizarre effects of Special and General Relativity can all be explained in some way by assuming that there is some fundamental reality which different observers interpret in different ways because of their different perspective or point of view. I have tried to show that this is fundamentally false. Indeed, the very principle on which Relativity is based is that every observer has an equal right to say that what he sees is reality.

The logical consequence of this, however, is that different observers experience different realities. Ludvig sees the rope break because it shrinks; Albert sees the rope break because the front spaceship started off before the other one. Who is right? They are both right. There is no higher dimension space which contains some ultimate reality. Obviously they must both see the rope break but the physics of the situation they experience is completely different.

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justification based on the formula $\gamma = 1 + gD/c^2$ which is the first order approximation to the above formula but the use of this formula is hard to justify as g can, in principle take any value and so the higher order terms should not be ignored. I am not sure quite how to resolve this issue but I am convinced that in any realistic scenario such as one in which Albert accelerates at a modest rate and then decelerates rather than travelling at constant speed and then suddenly turning round, the General Relativistic treatment will give the same answer as the standard one.

Once you have rejected the 'illusionist' approach, it becomes necessary to find physical explanations for length contraction and time dilation. It turns out that there are two completely different processes which are needed to account for length contraction which apply to two completely different situations: when Albert shoots past Ludvig in a fast spaceship, Ludvig sees Albert's spaceship contracted by a factor $\gamma = \frac{1}{\sqrt{1 - v^2/c^2}}$ because the electromagnetic forces which hold his ship together are distorted by the motion of the ship through (Ludvig's) space; when Albert accelerates up to speed, the distance between Earth and his destination shrinks by a factor $\gamma = \frac{1}{\sqrt{1 - v^2/c^2}}$ because of the curvature of spacetime during the period of acceleration. Now it may seem remarkable – not to say miraculous – that two so totally different mechanisms should result in the same expression for the contraction. So much so, in fact, that you may be tempted to conclude that I have got it all wrong and that there must be a single process that explains both effects. This is not so. It is a necessary consequence that the two processes should result in the same expression otherwise a logical inconsistency would result. Consider the broken rope paradox again, this time from the point of view of one of the pilots in the two spaceships. As usual, we shall take the view that he is stationary the whole time. When he fires up his rockets he considers himself to be in a gravitational field holding him still. He also knows that his friend in the other spaceship is also firing his rockets and therefore staying still too. But the rope is not so constrained and is taking a different route through spacetime and like the distance between Earth and Alpha Centauri, when the rockets cease firing, the two ends of the rope end up by being closer together than they were – in other words, the rope has to part company with one or other of the ships. Now if the contraction according to General Relativity was different from the contraction due to Special Relativity, then there would be a logical inconsistency between what Ludvig sees and what the pilot sees. This simply cannot happen. This is why I am so confident that, even though I know nothing about the mathematical structure of General Relativity, it must predict exactly the same degree of contraction.

What about the two effects of time dilation? Here we do seem to have two completely different processes and two apparently different formulae. Clocks moving through (Ludvig's) space at a speed v slow down by a factor $\gamma = \frac{1}{\sqrt{1 - v^2/c^2}}$, clocks at the bottom of a mine of depth D in a gravitational field of g slow down by a factor $\gamma = \frac{1}{\sqrt{1 - 2gD/c^2}}$ compared with a clock at the top. The explanation for the first effect is to be found in the constancy of the speed of light which, like the explanation for the shrinking of moving rulers, is based on Maxwell's Lorentz invariant laws of electromagnetism. The second effect, like the contraction of the distance between Earth and Alpha Centauri, is due to the curvature of spacetime by gravity. But as with length contraction, if Albert, using the General Relativistic formula for gravitational Time Dilation were to get a different result than Ludvig, using the Special Relativity formula, a logical inconsistency would result. I am confident that this does not happen.

So both length contraction and time dilation exhibit complementary processes which dovetail together in such a way that the laws of physics correctly explain reality from every angle. How remarkable is that?